Confidence Intervals and Hypothesis Tests: Two Samples

9.11 Hypothesis Test to Compare Two Population Variances

1. The U.S. census bureau released earnings data for year-round full-time workers by age and educational attainment in 2007. The results showed that male workers who were 25 -34 years old with a bachelor's degree earned more than males in the same age group with just a high school diploma. A new study selected a random sample of 21 males (aged 25 – 34) with a bachelor's degree and found they had an average salary of \$66,825 with a standard deviation of \$6,684.5, and a random sample of 22 males with only a high school diploma had a mean of \$35,995 and a standard deviation of \$6,245.1. Use the data provided and a 1% significance level to test the claim that earnings for males with a bachelor's degree have a greater variance than the earnings for males with just a high school diploma.

2. Due to historic discrimination of women in the workforce, a researcher in 2007 hypothesized that women in the 25 – 34 age group with a bachelor's degree would have a higher average income than women in the 35 – 44 age range with a bachelor's degree. A sample of 16 women in the 25 – 34 age group with a bachelor's degree had an average salary of \$49,966 with a standard deviation of \$5,104; while a sample of 15 women with a bachelor's degree in the 35 – 44 age group had a mean salary of \$32,358 with a standard deviation of \$4,995. At the 5% significance level, test the claim that the earnings for women in the 25 – 34 age group with a bachelor's degree have the same variance as earnings for women in the 35 – 44 age group with a bachelor's degree.

3. Average hourly earnings for workers have increased by about 50% since the 1990s, but that fact does not take into account inflation. Inflation matters because if your salary goes up by 50% while your cost of living rises by 100% you are losing ground. A study looked at this by randomly selecting wages for workers paid in 1970 and paid in 2008 and compared them after first converting them into constant (1982) dollars. A random sample of average hourly wages for 31 industries in 2008 found (in constant dollars) they had an average hourly wage of \$8.30 with a standard deviation of \$0.60. A random sample of 28 different industries' average hourly wages from 1970 had an average of \$8.46 (in constant dollars) with a standard deviation of \$0.55. At the 2.5% significance level test the claim that the hourly wages for workers in 2008 have greater variance than the wages for workers in 1970.

Answers:

1. Based on the sample data, it seems we cannot conclude that the variance for earnings of males in this age group with a bachelor's is higher than the variance for the earnings of males with only a high school diploma.

   $Claim: \sigma_1^2 > \sigma_2^2$

   $H_0: \sigma_1^2 \le \sigma_2^2$

   $H_a: \sigma_1^2 > \sigma_2^2$

   $n_1 = 21, s_1 = 6684.5$

   $n_2 = 22, s_2 = 6245.1$

   $Test\ Stat: F = \frac{6684.5^2}{6245.1^2} \approx 1.146$

   $Critical\ Value(s): f_{20,21,0.01} = 2.880$

   $Initial\ Conclusion:$ Do not reject the null, do not support the alternative

   $Final\ Conclusion:$ The sample data does not support the claim...

2. Based on the sample data, it seems that it is safe to assume that the two groups have the same variance.

   $Claim: \sigma_1^2 = \sigma_2^2$

   $H_0: \sigma_1^2 = \sigma_2^2$

   $H_a: \sigma_1^2 \ne \sigma_2^2$

   $n_1 = 16, s_1 = 5104$

   $n_2 = 15, s_2 = 4995$

   $Test\ Stat: F = \frac{5104^2}{4995^2} \approx 1.044$

   $Critical\ Value(s): f_{15,14,0.025} = 2.95$

   $Initial\ Conclusion:$ Do not reject the null, do not support the alternative

   $Final\ Conclusion:$ The sample data does not allow rejection of the claim...

3. The sample data does not provide strong enough evidence to conclude that wages from 2008 have a greater variance than wages from 1970.

   $Claim: \sigma_1^2 > \sigma_2^2$

   $H_0: \sigma_1^2 \le \sigma_2^2$

   $H_a: \sigma_1^2 > \sigma_2^2$

   $n_1 = 31, s_1 = 0.60$

   $n_2 = 28, s_2 = 0.55$

   $Test\ Stat: F = \frac{0.60^2}{0.55^2} \approx 1.190$

   $Critical\ Value(s): f_{30,27,0.025} = 2.13$

   $Initial\ Conclusion:$ Do not reject the null, do not support the alternative

   $Final\ Conclusion:$ The sample data does not support the claim...